Solution to Exercise 9

1. Suppose x^* is a global minimum of f. Let $x \in \mathbb{R}^n$ and $x \neq x^*$. Then

$$f\left(x^*\right) \leq f\left(\frac{1}{2}\left(x+x^*\right)\right) < \frac{1}{2}\left(f(x)+f\left(x^*\right)\right).$$

Hence, $f\left(x^*\right) < f(x)$. So x^* is the unique global minimum.

2. (a) Feasible set: $\{(1,0)\}$ Optimal solution: $x^* = (1,0)$.
   (b) KKT conditions:

$$\begin{aligned}
\left(x_1^*-1\right)^2+\left(x_2^*-1\right)^2 &\leq 1, \\
\left(x_1^*-1\right)^2+\left(x_2^*+1\right)^2 &\leq 1, \\
\lambda_1^*\left(\left(x_1^*-1\right)^2+\left(x_2^*-1\right)^2-1\right) &= 0, \\
\lambda_2^*\left(\left(x_1^*-1\right)^2+\left(x_2^*+1\right)^2-1\right) &= 0, \\
x_1^*+\lambda_1^*\left(x_1^*-1\right)+\lambda_2^*\left(x_1^*-1\right) &= 0, \\
x_2^*+\lambda_1^*\left(x_2^*-1\right)+\lambda_2^*\left(x_2^*+1\right) &= 0
\end{aligned}$$

Since $x_1^*=1, x_2^*=0$, we have

$$1+\lambda_1^*(1-1)+\lambda_2^*(1-1)=0$$

Hence, there is no λ_1^*, λ_2^* that satisfy the above equation.

3. See Page 44 of Note 1.